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Marangoni Convection Boundary Layer Flow of Ferrofluid with the Effects of Thermal Radiation and Suction

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ABSTRACT

The problem of thermal Marangoni convection boundary layer flow in ferrofluid with the presence of radiation and suction is evaluated in this paper. This study considered magnetite ferroparticle (Fe_3O_4) in two different base fluids, which are water and kerosene. The method of the solution involves similarity transformation, which reduces the governing partial differential equations to a system of an ordinary differential equation. Then, the equations are solved utilizing built-in bvp4c function in MATLAB software. The impacts of the suction parameter and thermal radiation parameter concerning the velocity profiles, temperature profiles, surface velocity and heat transfer are illustrated in graphs and tables. It is revealed that the suction parameter reduces the velocity profiles, temperature profiles and surface velocity. However, the heat transfer increases with the suction parameter; meanwhile, the thermal radiation parameter increases the temperature profiles. However, it reduces heat transfer. The same trends are observed in both water and kerosene ferrofluid. Nevertheless, higher heat transfer is observed in kerosene-based ferrofluid.

Keywords: Marangoni convection, boundary layer, ferrofluid

1. INTRODUCTION

Marangoni convection is the convection induced by varieties of surface tension gradients. The variations of temperature surface tension rise the thermal Marangoni convection while variations of concentration surface tension produce the solutal Marangoni convection, as mentioned by Al-Mudhaf and Chamkha [1]. The study of Marangoni boundary layers has attracted massive interest among researchers, where there have been numerous studies to investigate the Marangoni boundary layers. This growing interest is motivated by the essential applications of Marangoni boundary layers in the industrial process. Napolitano [2] appears to be the first who presented the Marangoni boundary layer term after studying the presence of the consistent dissipative layers occurring along the liquid-gas or liquid-liquid interface. Dissipative layers play essential roles in liquid metal and semiconductor processing and have become the primary consideration in directing the control of industrial processes, as stated by Chamkha et al. [3]. Furthermore, Abdullah and Lindsay [4] found that Marangoni convection can occur in a nanofluid due to the presence of the nanoparticles, which changes the surface tension of the base fluid.

Moreover, Arifin et al. [5] presented the analysis of the Marangoni convection boundary layer flow problem in nanofluids. The authors concluded that nanoparticles having low thermal conductivity such as titanium dioxide possess more considerable improvement on heat transfer contrasted to copper and aluminium oxide. Then, Hamid et al. [6] continued the work of Arifin et al. [5] by studying the radiation effect on Marangoni convection flow in nanofluids. The results revealed the radiation parameter's effects on heat transfer characteristics. It is noticed that for any nanoparticles category, an increase in the radiation parameter decreases the surface temperature gradients.

On the other hand, magnetic nanofluids are also called ferrofluids. Rashad [7] found that ferrofluids are magnetic nanoparticles' colloidal suspensions distributed in a non-conductive liquid. This particular type of nanofluids can be fundamentally utilized to manage the fluid flow and the rate of heat transfer, as stated by Ilias et al. [8]. Ferrofluids are often used in high-temperature devices, which leads to the consideration of the thermal radiation's effect in this research. Also, Titus and Abraham [9] analyzed the impact of radiation on the heat transfer in ferrofluid flow over a stretching sheet. The unpredictable change in temperature or the rapid stretching of the extrude might damage the anticipated characteristics of the final product. Thus, the heat transfer rate should be controlled cautiously. The result discovered that radiation could significantly affect to regulate the heat transfer's rate in the boundary layer area.

Additionally, investigation on stagnation point flow and heat transfer toward a stretching sheet with the occurrence of viscous dissipation had been done by Khan et al. [10]. The result yielded that water-based ferrofluids possess lower skin friction and Nusselt numbers compared to kerosene-based ferrofluids. This problem was extended by Khan et al. [11] with the slip effect and concluded that the slip parameter reduced the skin friction, and kerosene-based magnetite gives higher heat transfer rate at the wall than kerosene and Mn-Zn ferrite-based cobalt ferrite.

Meanwhile, this research also corporates the effects of fluid suction, which is a valuable strategy to avoid boundary layer separation. Suction is often essential to delay separation of the boundary layer to minimize the drag force. Kasmani et al. [12] found that the effect of suction can significantly change the rate of heat mass transfer. The suction process has its importance in numerous engineering activities, such as to control the boundary layers on the surfaces of aircraft wings or turbine blades, fluid filtration processes and thrust bearing design, as mentioned by Khaled [13]. Due to various applications and the significance of the factors discussed above, this study attempts to investigate the Marangoni convection boundary layer flow of ferrofluid corresponding to the effects of thermal radiation and suction. Furthermore, Mahdy and Ahmed [14] studied the thermosolutal Marangoni boundary layer magnetohydrodynamic flow in the regular fluid. Therefore, this research intends to extend the problem to thermal Marangoni convection boundary layer flow in ferrofluid corresponding to the effects of thermal radiation and suction. Note that Tiwari and Das [15] model has been considered in this research.

2. MATHEMATICAL FORMULATION

In this study, we examine a steady, two-dimensional and laminar Marangoni boundary layer flow of an incompressible ferrofluid as shown in Figure 1. The surface is assumed to be permeable and the wall mass suction velocity is denoted by v_w. The surface tension σ responsible for Marangoni convection is defined to differ linearly with temperature as given by (see Mahdy & Ahmed [14])

$$\sigma = \sigma_0 - \gamma(T - T_\infty) \tag{1}$$

where σ_0 is denoted as the surface tension at the interface, while γ refers to the surface tension's rate of change corresponding to the temperature. It is, therefore, assumed that a constant magnetic field, B_0 is applied to the surface in the normal direction, and that radiative heat flux is in the y-direction.

This study also considers two base fluids, namely water or kerosene based ferrofluid, which contains magnetite (Fe_3O_4) ferroparticle. In addition, the base liquid and the particles are assumed to be in thermal equilibrium, so no slip between them occurs. Cartesian coordinate system is denoted as (x, y), where x and y are the coordinates determined along the interface, and the flow happens at $y \geq 0$, are also considered normal. The governing equation for the conservation of this problem in mass, momentum equation and energy equations can be indicated as (see Mahdy & Ahmed [14]; Sastry et al. [16])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{ff}}{\rho_{ff}}\frac{\partial^2 u}{\partial y^2}-\frac{\delta_{ff}}{\rho_{ff}}B_0^2 u \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{ff}\frac{\partial^2 T}{\partial y^2}-\frac{1}{(\rho c_p)_{ff}}\frac{\partial q_r}{\partial y} \tag{4}$$

where u and v denote the components of velocity along the x and y-axes. Also, T denotes the temperature, μ_{ff} refers to the effective viscosity of ferrofluid, ρ_{ff} denotes the effective density, δ_{ff} denotes the electrical conductivity, B_0 refers to the uniform magnetic field, α_{ff} denotes the thermal diffusivity, q_r refers to the radiative heat flux and $(\rho c_p)_{ff}$ denotes the heat capacity of the ferrofluid.

The suitable boundary conditions are expressed by:

$$\left.\begin{array}{l} v=v_w,\ T=T_\infty+ax^2,\ \mu_{ff}\dfrac{\partial u}{\partial y}=\dfrac{d\sigma}{dT}\dfrac{\partial T}{\partial x}\ \text{ at } y=0 \\ u=0,\ T=T_\infty\ \text{ as } y\to\infty \end{array}\right\} \tag{5}$$

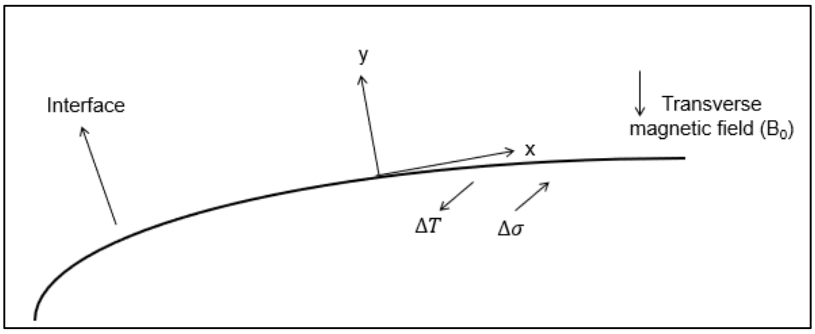


Figure 1. Graphical diagram of the problem.

3. SIMILARITY TRANSFORMATIONS

To obtain the similarity solutions for equations (2)-(4) with boundary conditions (5), the following similarity variables have been introduced in the form below (see Mahdy & Ahmed [14])

$$\psi = C_1 x f(\eta),\ \eta = C_2 y\ ,\theta(\eta) = \frac{T - T_\infty}{ax^2} \tag{6}$$

$$C_1 = \left(\frac{a\gamma \nu_f}{\rho_f}\right)^{1/3},\ C_2 = \left(\frac{a\gamma}{\mu_f \nu_f}\right)^{1/3}$$

where C_1 and C_2 are the similarity transformation coefficients and ψ denotes the stream function expressed as:

$$u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x} \tag{7}$$

Then, μ_{ff}, ρ_{ff} and α_{ff} are defined as follows (see Khan et al. [10]; Rashad [7])

$$\begin{aligned}
&\mu_{ff} = \frac{\mu_f}{(1-\phi)^{2.5}},\ \ \rho_{ff} = (1-\phi)\rho_f + \phi\rho_s,\\
&\alpha_{ff} = \frac{k_{ff}}{(\rho c_p)_{ff}},\ (\rho c_p)_{ff} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s,\\
&\frac{k_{ff}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)},\\
&\frac{\delta_{ff}}{\delta_f} = 1 + \frac{3(\frac{\delta_s}{\delta_f} - 1)\phi}{(\frac{\delta_s}{\delta_f} + 2) - (\frac{\delta_s}{\delta_f} - 1)\phi}
\end{aligned} \tag{8}$$

with Rosseland approximation $q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$ and $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$.

Substituting equations (6)-(8) and Roseland approximation into equations (2)-(4), while equation (2) is identically satisfied, we obtain

$$\varepsilon_1 f'''(\eta) + f(\eta)f''(\eta) - f'^2(\eta) - \varepsilon_2 M^2 f'(\eta) = 0 \tag{9}$$

$$\frac{1}{P_r}\frac{\varepsilon_3}{\varepsilon_4}[1 + N_r]\theta''(\eta) + f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta) = 0 \tag{10}$$

where M is the Hartmann number, $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and ε_2 are defined as follows:

$$M = \frac{\delta^{1/2} B_0 (\mu_f)^{1/6}}{(\rho_f a\gamma)^{1/3}}, \varepsilon_1 = \frac{1}{(1-\phi)^{2.5}\left((1-\phi) + \phi\frac{\rho_s}{\rho_f}\right)}, \varepsilon_2 = \frac{\delta_{ff}/\delta_f}{(1-\phi) + \phi\frac{\rho_s}{\rho_f}}, \varepsilon_3 = \frac{k_{ff}}{k_f}, \varepsilon_4 = (1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}$$

The boundary conditions (5) then becomes

$$f(0)=s,\ \frac{1}{(1-\phi)^{2.5}}f''(0)=-2,\ \theta(0)=1 \qquad (11)$$
$$f'(\infty)\to 0,\ \theta(\infty)\to 0$$

The physical quantities of interest in this research are the surface velocity, $u(x,0)$ and local Nusselt number, Nu_x expressed as

$$u(x,0)=u_w(x),\ Nu_x=\frac{xq_w}{k_f(T_w-T_\infty)} \qquad (12)$$

where u_w is the surface velocity and q_w is the surface heat flux, expressed as:

$$q_w=-k_{ff}\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (13)$$

By substituting Equation (6) into (12) and using equation (13), the following expressions can be attained

$$u_w(x)=C_1C_2xf'(0) \qquad (14)$$

$$Nu_x=-\frac{k_{ff}}{k_f}C_2x\theta'(0) \qquad (15)$$

4. RESULT AND DISCUSSION

The method of this study is validated by comparison with the results by Mahdy and Ahmed [14] for the case when the suction and ferrofluid parameters are absent $(s=0$ and $\phi=0)$. As seen in Table 1, the comparisons are considered to be in a rather close agreement. Consequently, we are confident enough of the numerical results reported.

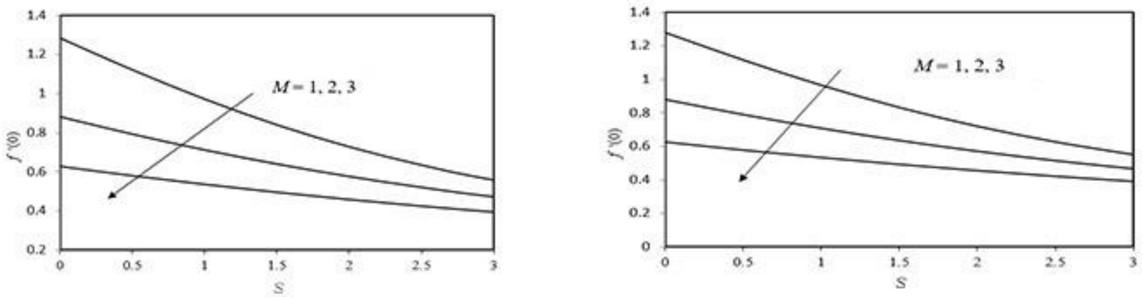
Table 1 Comparison for the $f'(0)$ values for various M when Pr =6.2 and $\phi = 0$.

$M = 0$		$M = 1$		$M = 2$	
Mahdy and Ahmed [14]	Present	Mahdy and Ahmed [14]	Present	Mahdy and Ahmed [14]	Present
2.519945	2.519730	2.226772	2.226767	1.6785735	1.678574

The parameters' values are chosen as follows; suction parameter (s = 1), the ferroparticle volume fraction (ϕ = 0.01), Hartmann number (M = 1), Prandtl number (Pr = 6.2 for water-based, Pr = 21 for kerosene-based) and thermal radiation parameter (Nr = 1).

a. Results on Fluid Flow and Surface Velocity

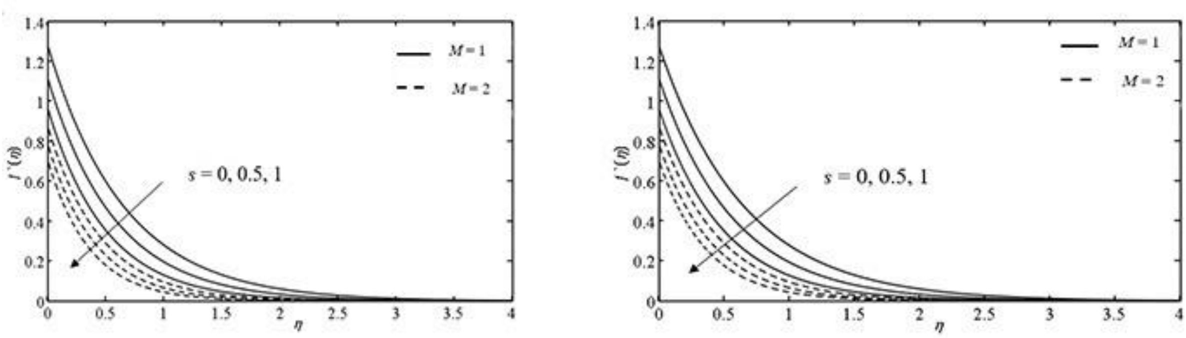
The surface velocity's variations, $f'(0)$ with respect to the suction parameter, s for various Hartmann number, M in water-based and kerosene-based ferrofluid are shown in Figure 2a) and 2b), respectively. It can be perceived that fluid suction reduces the surface velocity for both water-based and kerosene-based ferrofluid. When the fluid suction is presence at the surface, more fluids are brought into the surface, which reduces the velocity at the surface. Further reduction is noticed when magnetic field is imposed.



a) Water-based ferrofluid b) Kerosene-based ferrofluid

Figure 2. Variations of $f'(0)$ with s for various M when $\phi = 0.01$

The fluid suction's effect on the velocity profiles in water-based and kerosene-based ferrofluid is depicted in Figures 3a) and 2b), respectively. Both figures show that parameter s reduces the velocity profiles for both water and kerosene base ferrofluid. Further reduction of the surface velocity and the velocity profiles can be seen when parameter M is present, as shown in Figures 2-3. This phenomenon is due to the presence of the Lorenz force when parameter M increases. The force provides more resistance to the fluid transport.

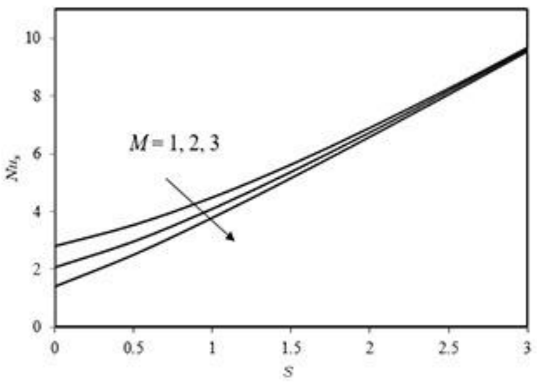


a) Water-based ferrofluid b) Kerosene-based ferrofluid

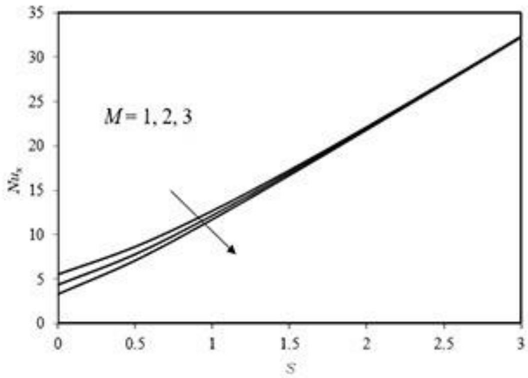
Figure 3. Effects of suction parameter, s on velocity profiles for various M

b. Results on Fluid Temperatures and Heat Transfer

The variations of heat transfer represented by Nusselt number, Nu_x with parameter s are depicted in Figures 4a)-4b) for water-based and kerosene-based ferrofluid, respectively. The numerical values are tabulated in Table 2. From the figures and table, it can be seen that fluid suction increases the heat transfer for both water-based and kerosene-based ferrofluid. This is due to a reduction in the thickness of the boundary layer when the suction of fluid is present at the surface. Reduction in boundary layer thickness may intensify the heat transfer. Nevertheless, the presence of the suction parameter reduces the temperature profiles, especially in the kerosene-based ferrofluid, as shown in Figure 5a) and 5b). This is also due to a reduction in boundary layer thickness.



a) Water-based ferrofluid



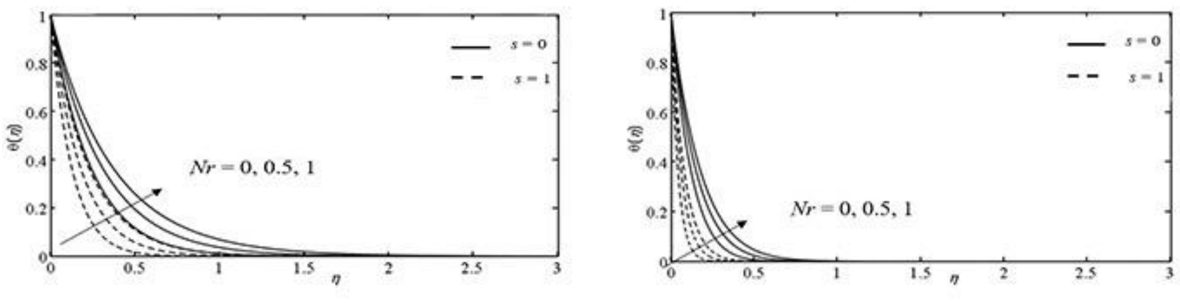
b) Kerosene-based ferrofluid

Figure 4. Variations of Nu_x with *s* for various values of *M* when $\phi = 0.01$

Table 2 Variations of Nusselt number with thermal radiation parameter, *Nr* for various parameters *M* and *s*.

		$M = 0$			$M = 1$	
s	*Nr* = 0	*Nr* = 0.5	*Nr* = 1	*Nr* = 0	*Nr* = 0.5	*Nr* = 1
Water based ferrofluid						
0	4.679972	3.759802	3.210681	4.154188	3.315880	2.815189
0.5	6.263012	4.728968	3.885648	5.824832	4.351751	3.546041
1	8.303063	5.972965	4.751669	7.993428	5.696803	4.498048
Kerosene based ferrofluid						
0	8.948830	7.246573	6.231532	8.044879	6.495142	5.570821
0.5	15.508377	11.383195	9.203866	14.831727	10.794962	8.673405
1	24.055975	16.768480	13.063619	23.648369	16.392715	12.711724

The effect of the thermal radiation on water and kerosene base ferrofluid temperature profiles is illustrated respectively in Figures 5a) and 5b). Both figures show that the thermal radiation increases the temperature profiles for both water and kerosene base ferrofluid. From Table 2, it can be observed that the heat transfer is reduced with an increase in the parameter *Nr*. Increasing the thermal radiation increases the fluid temperature and eventually reduces the heat transfer at the interface. However, the occurrence of the suction parameter decreases the temperature profiles, especially in kerosene-based ferrofluid.



a) Water-based ferrofluid   b) Kerosene-based ferrofluid

Figure 5. Effects of radiation parameter, *Nr* on temperature profiles for different *s*

CONCLUSION

From the results, the suction parameter slows down the magnetite ferrofluid flow, and further reduction of velocity occurred when magnetic strength is increased. Effects of the parameter in water-based and kerosene-based show the same trend. The suction parameter also causes the surface velocity to reduce, where further reduction occurred when magnetic strength is increased.

In addition, the thermal radiation increases the fluid temperature. However, the suction parameter reduces the fluid temperature, especially in kerosene-based ferrofluid. Moreover, the suction parameter increased heat transfer. Nevertheless, the strong magnetic field causes the heat transfer to reduce, especially in kerosene-based ferrofluid.

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