

Analysis of structure reliability on beam using fuzzy finite element method (FFEM)

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Abstract: *The main requirement in designing a structure is to ensure the structure is reliable enough to withstand any loading. However, in the real world, for structural analysis, the presence of uncertainties in the input variable has reduced the accuracy of the calculated structural reliability. The purpose of this study is to determine the structural reliability with the consideration of uncertainties involved. The developed simulation method is the fuzzy set theory incorporating with the finite element methods followed with margin safety based on the yield strength of the structural reliability. This method is then used to analyze a given beam structure under loading for the material which are made from Aluminium 2024-T4. In this study, the modulus of section, s and loading, w are used as a fuzzy parameters. In conclusion, the combination of fuzzy set theory with the finite element method plays an important role in determining the structural reliability in the real world.*

Keywords: *Uncertainty, Fuzzy set theory, Finite element method, Reliability, Margin safety*

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1 Introduction

Structural reliability is an important factor for developing countries, especially on the industrial and technological equipment. Structural reliability can be defined as the ability of a structure to carry out its functions in a single lifetime, and under certain condition [1]. Lots of the planning and design of engineering structures should be completed without enough information. In this case, the design performance cannot be guaranteed.

Moreover, many decisions in the structure design process had to be performed in an uncertain situation. So, the chances of failure of a structure always exist and are unavoidable. This resulted in

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the design of a structure having the absolute security is not practical. Furthermore, in the structural reliability analysis, the uncertainty for analysis becomes a constraint. He et al. [2] mention that, in general, uncertainty can be divided into three types, which are stochastic uncertainty, epistemic uncertainty and error.

Stochastic uncertainty is due to variations in the system or environment. For the epistemic uncertainty, it exists as a result of incomplete information, ignorance and lack of knowledge caused by the lack of experimental data. When compare to the error, this uncertainty is the uncertainty that can be identified due to the imperfections in the modeling and simulation.

For a several decades ago, uncertainty is modelled according to the theory of probability [3]. According to [4] the basic methods in determining the structural reliability by applying the theory of probability is the Monte-Carlo Simulation (MCS), First Order Reliability method (FORM) and Second Order Reliability method (SORM). Probability method is very effective in solving the problem of stochastic uncertainty, but this method is not suitable to be used to solve problem involving the lack of data.

In reality, the information on load and structural properties are lacked and sometimes it is difficult to be obtained [5]. In addition, another aspect such as the relationship between the structure of the enviroment and human error cannot be effectively associated by using the probability methods. Some scholars hold that the use of non probability methods are most appropriate to interpret the an uncertainty compared to statistical approach when the deal with the lack of data. The interval analysis, convex modeling and fuzzy set theory are the main categories of non-probabilistic methods [6]. In this research, the combinations of fuzzy set theory with the finite element methods are developed to discuss about the accuracy and sensitivity of proposed approach through structural model irregular component which is made from Alumimun 2024-T4.

2 Methodology

In this research, the analysis on structural reliability in the presence of uncertainties is performed by using the combination of fuzzy and finite element methods. According to [7], the requirement for the accuracy and efficiency approach for assessing uncertainty in loading, geometry, material properties, manufacturing processes and operating environment have increased significantly. Besides, this part also discussed the fuzzy structural reliability, maximum normal stress and margin safety.

2.1 Fuzzy structural reliability definition

The definition of fuzzy reliability of the structure begins with the determination of safety margins, Z , center of gravity or centroid, COG and maximum normal stress, S_{max} . Given that the yield strength of the variable distribution is normal. So the probability is used in the calculation P_r with a mean value, m_{s_y} and standard deviation, σ_{s_y} is fixed. In this context, the probability of structural failure, P_f is the maximum normal stress exceeds the yield strength and can be represented by the following equation:

$$P_f = P(S_y \leq S_{max}) = \Phi \left(\frac{S_{max} - m_{s_y}}{\sigma_{s_y}} \right) \quad (1)$$

Then the reliability P_r can be evaluated using the following equation:

$$P_r = 1 - P_f \quad (2)$$

2.2 Maximum normal stress

Maximum normal stress S_{max} is a parameter that is commonly used in structural design. Internal force is the shear force, V has magnitude equal to the load P and bending moment, M . Bending moments are then creating the normal stress on the cross section, while the shear force, V produces shear stress on the section. Maximum normal stress depends on the bending moment and section modulus, S and then:

$$S_{max} = \left| \frac{M_{max}}{s} \right| \tag{3}$$

and

$$s = \frac{I}{c} \tag{4}$$

where c is the maximum distance from the neutral surface and I is the moment of inertia of the cross-section. The bending moment equation for a uniform load beam and a simple support beam is shown below:

$$M = -\frac{w}{8} (4x^2 - 5lx + l^2) \tag{5}$$

which w , x and l are represented loading, distance of the beam and the length the of beam respectively.

2.3 Margin safety

One way can be used to determine the probability of structure failure is by using margin safety approach. This approach can be used the equation below:

$$Z = Q - S \tag{6}$$

where Q is the resistance of the system and S is the effect of the applied load, Z is a function in terms of Q and S . If $Z < 0$, the system is in the region of failure and if $Z > 0$, the system is said to be in the safe region. But if $Z = 0$, is known as the failure surface [3].

2.4 Fuzzy set theory

The word fuzzy was introduced by Prof. Lotfi A. Zadeh in 1962 [8]. In 1965, he developed the concept of fuzziness in the paper by introducing the theory of fuzzy sets. The word fuzzy mean "vagueness" and fuzziness occur when the boundary of a piece of information is not clear enough. Today, the theory of fuzzy sets has applied in many areas of sciences and engineering.

A fuzzy set is a set that has no clear boundaries. Significantly different from the sets, fuzzy sets allow the transition from the status of "membership" to a status of "non-membership". In fuzzy set theory, fuzzy set A in X is represented by a universal function $\mu_A(x)$ is known as the membership function set A . There are several methods to fuzzified the variables which have been proposed by researchers. Some of the methods are intuition, inference, genetic algorithm, neural networks, inductive discretion, and the distribution of soft and fuzzy statistics [9]. The form of membership functions depends on the method used in the fuzzy process. The normal form and triangle form are

two of the most common form of membership function. Triangular membership functions are widely used to simplify the calculation. Figure 1 shows an example of fuzzy variables with triangular membership function and normal.

$$\mu_A(x) : X \rightarrow [0, 1] \quad (7)$$

which

$$\mu_A(x) = 1, \text{ if } x \text{ stay in } A \text{ region;}$$

$$\mu_A(x) = 0, \text{ if } x \text{ not in } A \text{ region;}$$

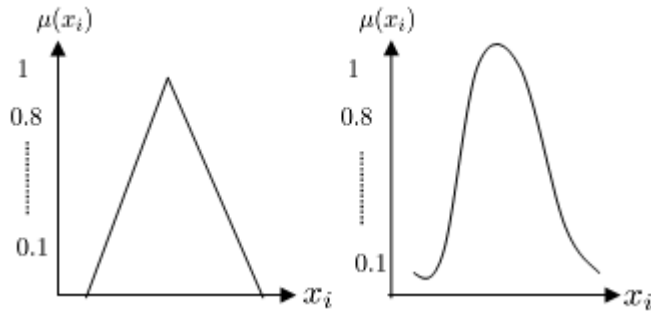


Figure 1: Membership function for triangular and normal form

2.4.1 Modelling of uncertainty fuzzy input

In the theory of fuzzy sets, [10] states that the human ability will decrease in making the right decision because of the existing of the uncertainty in the complex system. This uncertainty can easily express in the linguistics variable form. This expression can be modeled by using fuzzy set theory in the form of membership function. For this study, the procedure in analyze the structure reliability by using fuzzy set theory with fuzzification of the uncertainty input and followed by the α -cut process, defuzzification and reliability assessment.

Fuzzification can be defined as a specification of a membership function process for a certain uncertainty set [11]. In this context, each uncertainty parameter which is interpreted by a membership function which would give to a trend value.

2.4.2 Fuzzification of uncertainty input

Fuzzification can be interpreted as a specification of the membership function $\mu(x)$ of a set of uncertainty. The uncertainty of each parameter that is interpreted by a membership function will bring value to the trend. Two parameters of the modulus of section, s and loading, w are used as fuzzy parameters. Fuzzy normal stress is the result after the α -cut. It was found that, the normal stress depends on and bending moment and modulus of section, while the bending moment depends on the load. Therefore, the concept of α -cut corresponds to an aspect of the real world problem. At least, this kind of abstraction has ignored the symmetry of abstraction, and lost some information of the real world problem. Since both loading and bending moment is the dependent variable, then the bending moment is also classify as a fuzzy parameter. Triangular fuzzy numbers are used for understanding the function of all parameters of fuzzy membership. Upper limit value (s_u, w_u, M_u) and the lower limit (s_1, w_1, M_1) of these functions will be determined by expert opinion. While the middle value (s_t, w_t, M_t), is between the upper and lower limit. Then the two parameters are mapped to a decision (output) with the α -cut.

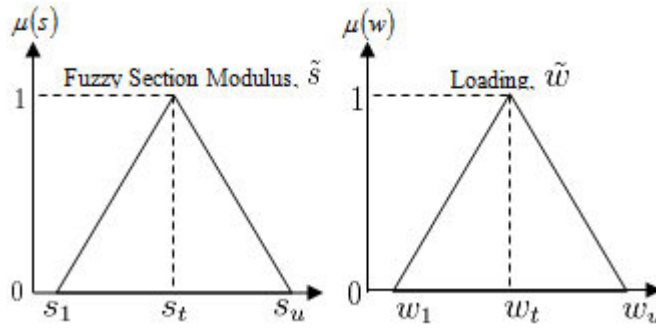


Figure 2: Membership functions with triangular fuzzy number for the modulus of the section and loading

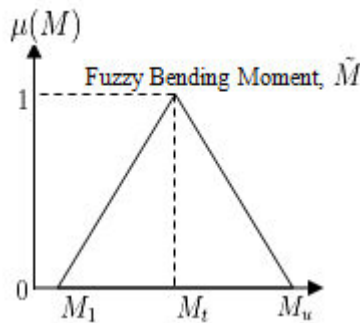


Figure 3: Membership function with triangular fuzzy numbers of bending moment

Figs. 2 and 3 shows that the tendency for the upper limits of the bending moment, M_1 and the lower limit M_u is zero, ie no tendency for the moment, whereas the trend with a value of 1 implies that the trends of the moment is the best, and symbolized by the symbol M_t .

2.4.3 α - cut process

The α -cut is one way of mapping that maps fuzzy input to fuzzy output with specific functions (relationship between two or more entities) as show in Figure 4. Mapping of the input (modulus of section, s and the bending moment, M) to output (the maximum normal stress, S_{max}) performed. The term mapping is specified here to mean logical after the fuzzification. In this process, all the fuzzy section modulus, \tilde{s} and fuzzy bending moment, \tilde{M} at each stage of the trend, α mapped by using equations (3). Thismapping resulted in four significant values for each level of the trend, α at the normalstress space, S . The maximum and minimum values are select from a combination of these results and used as the upper limit and lower limit for the output fuzzy.

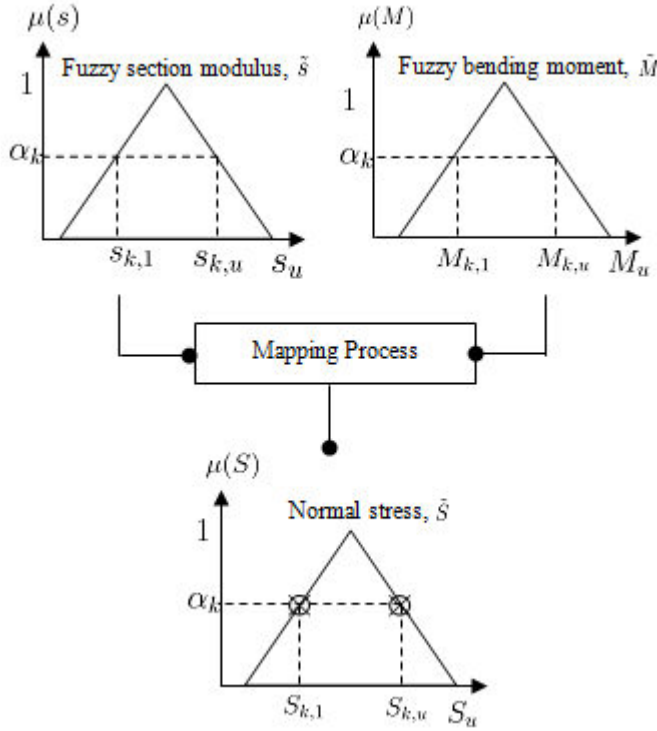


Figure 4: Mapping of \tilde{s} and \tilde{M} at all stages of the trend α

2.4.4 Defuzzification of normal stresses

Defuzzification of normal stress is using the center of gravity or centroid. This technique determines the point at which it will distribute one area (area graph) into two parts which have the same value. Mathematically, the point is called the center of gravity, COG [12]. COG is expressed as equation (8).

$$COG = \frac{\int_{S_1}^{S_u} S \mu_s(S) dS}{\int_{S_1}^{S_u} \mu_s(S) dS} \tag{8}$$

Using trapezoidal rule, the equation of the new COG would be formed as shown in equation (9).

$$COG = \frac{\sum_{i=1}^{n-1} (S_0) - \frac{1}{2}(\mu_i + \mu_{i+1})(S_{i+1} - S_i)}{\frac{h}{2} \left[(\mu_{s_i} + \mu_{s_u}) + 2 \sum_{i=1}^{n-1} \mu_i \right]} \tag{9}$$

where

$$S_0 = \frac{3(\mu_i + \mu_{i+1})S_i + (\mu_i + 2\mu_{i+1})h}{3(\mu_{i+1} + \mu_i)} \tag{10}$$

2.5 Finite Element Method (FEM)

The finite element method (FEM) has been one of the most powerful numerical tools. The inspiration for the finite element method (FEM) comes the scholars to solve the complex structural analysis. FEM method was introduced by Alexander Hrennikoff in 1941 and Richard Courant in 1942. Olgierd Cecil Zienkiewicz is the leading one of the pioneers at that time to solve the problem other than solid mechanics using FEM. In the 1970s, FEM began applied to nonlinear problems also for large deformation. In this contemporary era, advanced performance computing industry has facilitated the scholars solve various problems by using FEM. FEM method is a numerical technique which consists of partial differential equations and integral equations. The pre-processing of FEM method consists of, the geometry, boundary conditions and loading conditions. Functions of the general solution of FEM methods are as follows:

$$k.u = F \tag{11}$$

In which F is the global load vector, k is the vector of global strength and u is the displacement vector. The post-processing of FEM method consists of displacement, stress, strain and so on. For problems with complex geometry and general boundary and loading conditions, to get a solution is a difficult task.

2.6 Fuzzy Finite Element Method (FFEM)

The fuzzy finite element method is a combination of fuzzy set theory and finite element method. Usually, the FFEM method is used when insufficient experimental data or no data. In this study, an isosceles triangular model is used as the membership functions for fuzzy input. Basically, the uncertainties of the parameters such as geometry, material and loading are used as a fuzzy input in FFEM method [13]. Fig. 5 shows a flow chart for a structural reliability analysis by using the fuzzy finite element method. In this study, the mapping function is based on the finite element method.

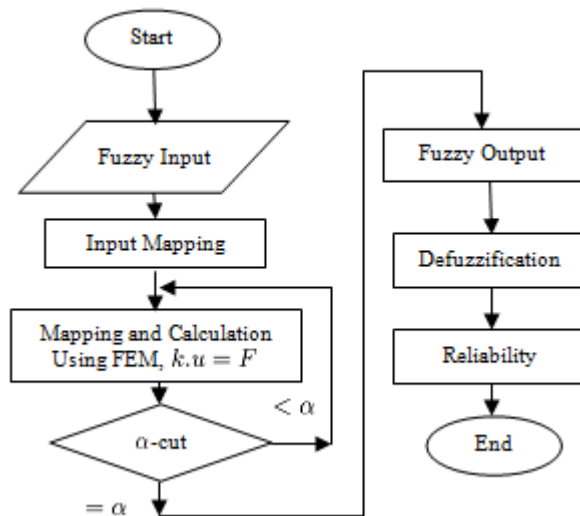


Figure 5: Flow chart for FFEM

The final fuzzy output of the FFEM simulation method is stress structure and this parameter still fuzzy until at the end of the process. The defuzzification process is continuing by applying the

center of gravity (COG), where the calculation of this method is described before. After the defuzzification process, the crisp stress structure value is obtained and this value is compared with the material to get the probability of failure and finally the reliability structure value.

2.6.1 Formulation of FEM

In the formulation of the finite element method for the beam, the beam is divided into smaller elements as shown in Figure 6. Each node has two degrees of freedom to which degree of freedom for node i is Q_{2i-1} and Q_{2i} . Degree of freedom for Q_{2i-1} is the transverse displacement and the degree of freedom for Q_{2i} is a rotation.

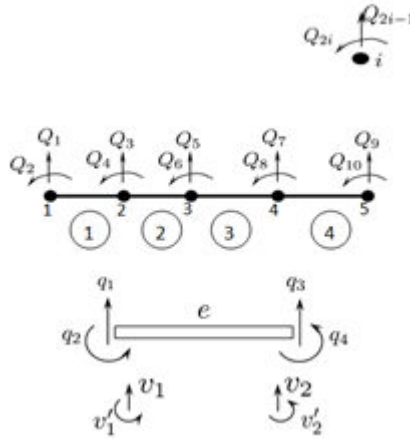


Figure 6: Discrete finite element

Vector Q is a global displacement vector like show in equation (12).

$$Q = [Q_1, Q_2, Q_3, \dots, Q_{2n}]^T \tag{12}$$

where, n = the number of node on beam. For some element, local degree of freedom is the presence in equation 13.

$$q = [q_1, q_2, q_3, q_4]^T \tag{13}$$

The relationship between the local displacement vector and global displacement vector can be seen from Fig. 6. In Fig. 6, q is equal to $[v_1, v_1', v_2, v_2']^T$. For the structure type of beam, the shape of the beam element is the Hermite shape. Hermite sharp functions are represented by the equation (14) which is third order equation.

$$H_i = a_i + b_i\xi + c_i\xi^2 + d_i\xi^3 \tag{14}$$

where $i = 1, 2, 3, 4$ and,

$$H_1 = \frac{1}{4}(1 - \xi)^2(2 + \xi)$$

$$H_2 = \frac{1}{4}(1 - \xi)^2(\xi + 1)$$

$$H_3 = \frac{1}{4}(1 + \xi)^2(2 - \xi)$$

$$H_4 = \frac{1}{4}(1 + \xi)^2(\xi - 1)$$

Stiffness matrix for the element of beam, k^e are defined in the form of below.

$$k^e = \frac{EI}{l_e^3} \begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix} \tag{15}$$

2.6.2 Striped solution

Striped solution method can be used to simply get the computation of the finite element method, especially if the global stiffness matrix is symmetric. If the global stiffness matrix ($n \times n$) in the form of Fig. 7, the element in the global stiffness matrix can be stored in the matrix ($n \times nbw$) as shown in Fig. 8. where the matrix in figure 8 is a partial bandwidth. Through this method, the only non-zero element should be retained.



Figure 8: Striped symmetry matrix

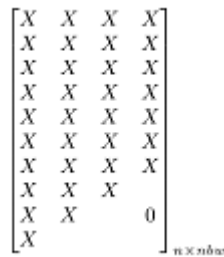


Figure 8: ($n \times nbw$) matrix

2.6.3 Shear force and bending moment

To determine the shear force and bending moment in finite element method, equation (16) and (17) as show below can be used.

$$V = \frac{6EI}{l_e^3} (2q_1 + l_e q_2 - 2q_3 + l_e q_4) \quad (16)$$

$$M = \frac{EI}{l_e^2} [6\xi q_1 + (3\xi - 1)l_e q_2 - 6\xi q_3 + (3\xi + 1)l_e q_4] \quad (17)$$

Shear force and bending moment for each element is determined by modelling the total of load on the element to an equivalent point load. To delegate the force on the end of the element as R_1 , R_2 , R_3 and R_4 , so the equation (18) are obtained.

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = \frac{EI}{l_e^3} \begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} + \begin{Bmatrix} \frac{-pl_e}{2} \\ \frac{-pl_e^2}{12} \\ \frac{-pl_e}{2} \\ \frac{pl_e^2}{12} \end{Bmatrix} \quad (18)$$

the shear force and bending moment at the both end of the element are $V_1 = R_1$, $V_2 = -R_3$, $M_1 = -R_2$ and $M_2 = R_4$.

3 Numerical result and discussion

The numerical results from this study by using the fuzzy finite element method (FFEM) are discussed below. In addition, analysis on sensitivity of a parameter also defined here. A study carried out should have the material, by the nature of the material in properties of density, strength and other should be known.

3.1 Aluminium 2024-T4 model

The model that used for this study is aluminium alloy 2024-T4. This material is commonly used to build gears and shafts, watch component, hydraulic valve parts, piston, and so forth. Table I describe the physical properties of the aluminium 2024-T4 model.

Table 1: Mechanical properties of alluminium 2024-T4

Physical Properties	SI unit
Density	2780 kg/m ³
Ultimate strength (Tension)	469 MPa
Yield Strength (Tension)	324 MPa
Elastic Modulus	73.1 GPa

3.2 Structural Model

The loading, w is 5.55 MN/m and moment of inertia, I is $6.58 \times 10^{-5} m^4$. The height of the beam structure is 0.161m and the structure is made of aluminium alloy 2024-T4 where the elastic modulus is 73.1 GPa. The statistical distribution of aluminium 2024-T4 is 324 MPa and 32.4 MPa which represent the mean value and standard deviation of the yield strength respectively as Fig. 9.

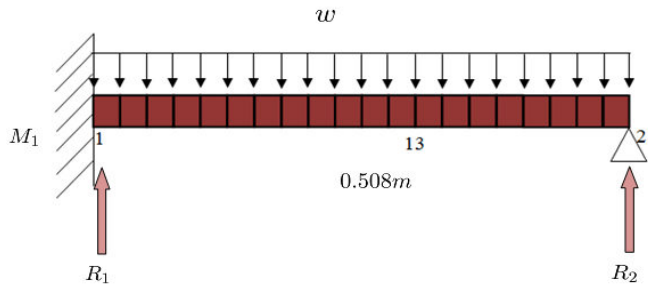


Figure 9: Position of the nodes on the beam structure from node 1 to node 21

3.3 Type of entry

There is a different type of entry for each method and is shown in Table 2.

Table 2: Type of entry

Method	Input Type	
	Uniform Distributed load	Moment of Inertia
Determinant	Average Value	Average Value
Stochastic	Normal distribution ^a	Normal distribution ^b
Fuzzy	Triangular Membership Function ^c	Triangular Membership Function ^c
FFEM	Triangular Membership Function ^c	Triangular Membership Function ^c

^a COV = 0.2

^b COV = 0.2

^c width = 6σ normal distribution

3.4 Fuzzy Finite Element Method (FFEM)

In general, there are three parameters that can be generated from the simulation developed FFEM. These three parameters are deflection, bending stress and bending moment. Simulations are carried out with α -cut is set to 10 and the beam structure was analyze by 20 elements. The result of this analysis is plotted on the 2-dimensional graph. The maximum value for each parameter can be determined from this graph.

3.4.1 Deflection

From the graph of deflection against the nodes in Figure 10 below, the highest possible deflection is located at node 13. For $\alpha = 0$ is represent the minimum and maximum, the $\alpha = 1$ is the highest confidence level. The red line graph represents the deflection for the COG. COG value is not equal to the value at the highest peak. In Fig. 11, the value of the maximum deflection of the beam structure is equivalent to the value of the COG deflection components at node 13 is 0.5mm.

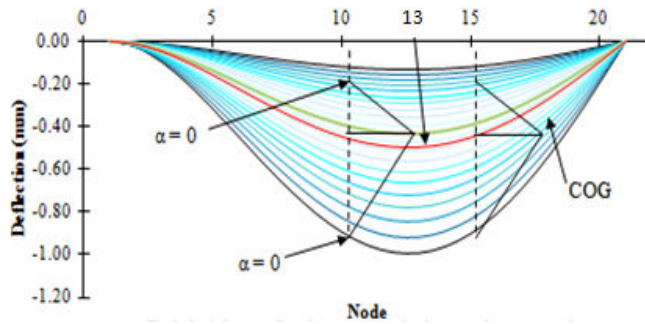


Figure 10: Fuzzy deflection against beam length

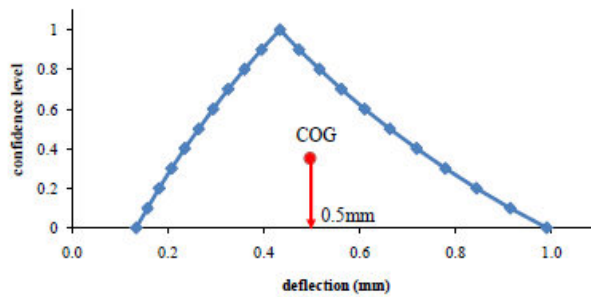


Figure 11: Maximum deflection of the fuzzy

3.4.2 Bending stresses

The stress of the beam structure is in the last stage of the fuzzy output of the FFEM simulation method in which the structure reliability is evaluated using the maximum value of stress obtained by COG. From Figure 12, it was found that the location of maximum stress analysis of the structure is at node 1, where the bending stress is 251 MPa as shown in Fig. 13. COG stress value is not equal to the value of stress on the value 1. These results indicate that, although the input profile is an isosceles triangle, but not necessarily the fuzzy output profile is an isosceles triangle. This statement concluded that the fuzzy output profile depends on a mapping function as well as fuzzy input profile.

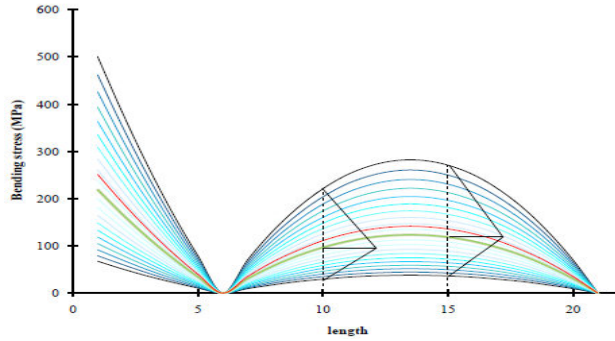


Figure 12: Fuzzy bending stress against beam length

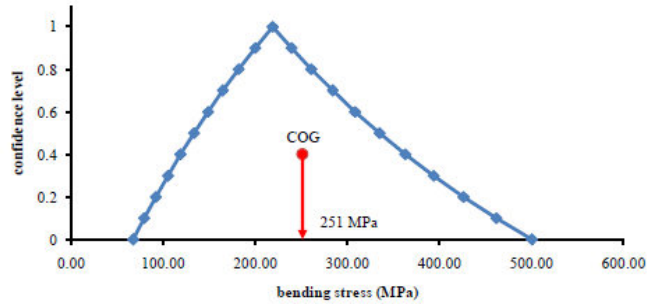


Figure 13: Maximum fuzzy bending stress

3.4.3 Bending moment

Fig. 14 shows the graph of bending moment against the node. The value of bending moment is highest at node 1 and this value is the equivalence of the COG value at node 1. Fig. 15 shows that, the fuzzy bending moment profile graph is an isosceles triangular. In addition, the simulation results proved that the value of fuzzy bending moment linearly with fuzzy input of loading. The COG value of the bending moment is 178.9 kN.m shown in Fig. 15.

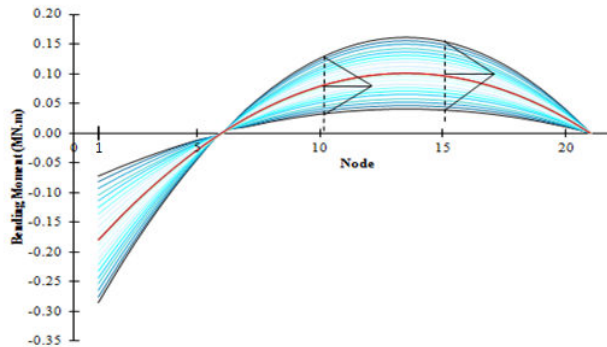


Figure 14: Bending moment graph against node

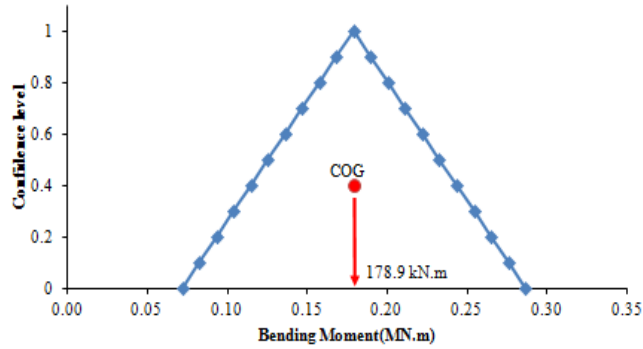


Figure 15: Maximum fuzzy bending moment graph

3.5 Analysis on sensitivity of parameter

The effects of parameters such as a number of node used in the calculation of the finite element method (FEM) and the number of α -cut of the reliability of the structure are discussed in this section.

3.5.1 Node numbers

Referring to Fig. 16, simulation FFEM with an element of reliability to the most different structure compared to the optimum value. It was found that the probability of failure approaches 0.052896 if more elements involve in the simulation of FFEM. This can be concluded that, the simulation FFEM method with three elements is enough to get the value of probability of failure.

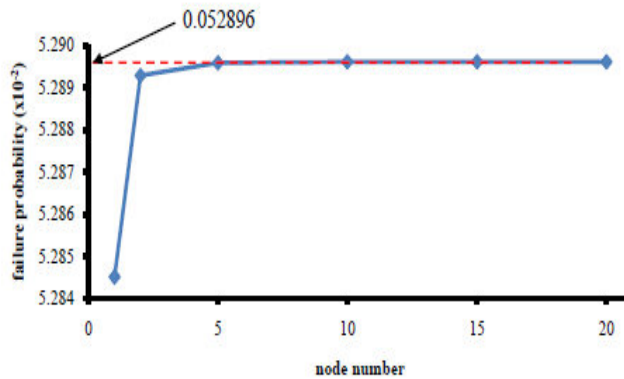
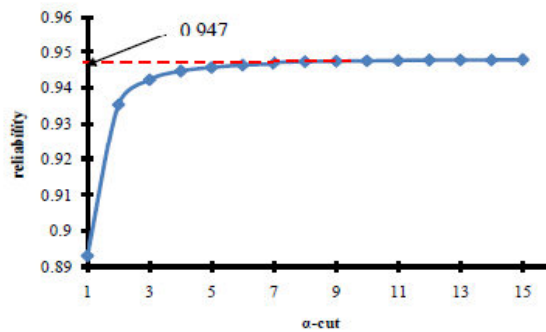


Figure 16: Failure probability against node number

3.5.2 α -cut

Fig. 17 shows the number of α -cut effect on reliability. It was found that the reliability of a structure is also approaching the optimal value like as the α -cut completed in the FFEM simulation. The optimum value of the reliability of structural analysis is 0.947 and this value starting focused on the α -cut number 10.

Figure 17: Reliability against α -cut

4 Conclusions

The FFEM approach is an effective method to analyze the structural reliability and probability of failure of a structure. This study shows that, the FFEM approach is most conservative compared to another method especially when involving the uncertainties, toward the reliability. One way to reduce the uncertainty in the data is by experiment. FFEM method developed does not require a large amount of data. FFEM method only required data to determine the profile or shape of membership function. The data can usually be obtained from the opinion of expert knowledgeable in the analysis associated with inductive reasoning or with a genetic algorithm. Modeling input in the form of membership function, effectively involving epistemic uncertainty in the analysis. Although both these methods based on fuzzy set theory, but the presence of a finite element method in FFEM approach allows us to use easily analyze the complex structure or component. By applying the FFEM approach, the increasing number of element and α -cut will lead to the accuracy value of reliability in this study. However, the efficiency of the simulation process will be limited if the number of element and α -cut is too much. This is because of our calculations would increase directly with increase in the number of element and α -cut. Thus, the dominant factor that affecting the efficiency and accuracy of the simulation is the number of element and α -cut used. In addition, the factor that affects the developed simulation of FFEM is the number of fuzzy parameters. The more obscure parameters involved in the simulation, the most conservative result of the analysis.

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